Probability, likelihood, sampling and expectation

Overview

- Probabilities and likelihood

- Sampling

- Linearity of expectation

What is a (discrete) distribution?

- Informal: bunch of positive numbers that some to one

- Distribution: $P : a \to [0,1]$

- Event: $a \in [0,\ldots,n-1]$

- Probability: $P(a)$

  - Chance of a occurring

What is a conditional probability?

- Conditional probability:

$P(a|\theta)$

  - Chance of a occurring given θ

- Example θ's:

  - Other event

  - Model parameters

What is the likelihood?

- Informal: same as probability

- Formal: a function of parameter θ that describes the probability of observing data x^n given θ.

- Definition: $\quad L(\theta) \equiv L(\theta; x^n) = P(x^n | \theta)$

- Usually refers to past events

Sampling

- Definition: $a \sim P$

  - Produce events a following $P(a)$

- Sampling bias

  - Empirical probability of samples != $P(a)$

Expectation

- Definition:

  - $\mathbb{E}_{a \sim P}[f(a)]$

- For any function f

  - $\displaystyle\sum_a P(a)f(a)$

  - $\displaystyle\frac{1}{N}\sum_{a \sim P} f(a)$

Linearity of expectation

- $\mathbb{E}_{a \sim P}[\alpha f(a)] = \alpha \mathbb{E}_{a \sim P}[f(a)]$

- $\mathbb{E}_{a \sim P}[f(a) + g(a)] = \mathbb{E}_{a \sim P}[f(a)] + \mathbb{E}_{a \sim P}[g(a)]$

- $\mathbb{E}_{a \sim P}[f(a)g(a)] \neq \mathbb{E}_{a \sim P}[f(a)]\mathbb{E}_{a \sim P}[g(a)]$

Summary

- Event: $a \in [0, \ldots, n-1]$

- Distribution: $P : a \to [0,1]$

- Probability: $P(a)$

- Sampling: $a \sim P$

- Expectation: $\mathbb{E}_{a \sim P}[f(a)]$

[Introduction to Probability, Bertsekas and Tsitsiklis 2002]
[All of Statistics, Wasserman 2004]